Pre-Algebra Notes

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This resource is pre-release and is still a work in progress. All contents are subject to change.

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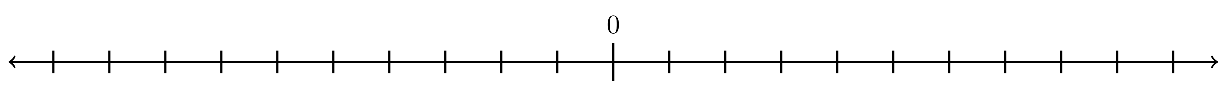
1.1 Integers and Absolute Value

The ______________________ are the numbers you can count to, starting from ________.

The _____________________ are the numbers you can count to, starting from ________ .

The _____________ are the numbers you can count to, but you're also allowed to count _______________ . This means the integers include the natural numbers and their ______________, as well as zero.

A ___________ number is any number ____________ than zero. A _____________ number is any number ________ than zero. A ___________________ in front of a number means that it has the ________________________ on a number line.



The ___________________ of a number is the ____________ of a number from zero on a number line. The symbol for absolute value is __________________ either side of a number.

Example

Evaluate each of the absolute value expressions.

$|7|$ $|-7|$ $|-4|$ $|9|$

We can use the symbols _____ (less than), _____ (greater than), and _____ (equals) to show the order of numbers. On a number line, lesser numbers are to the _______, and greater numbers are to the ________ .

Example

Write =, < or > to correctly indicate the order of each pair of integers.

$9 \quad 2$ $-4 \quad 1$ $3 \quad -8$

$5 \quad |-5|$ $-7 \quad -2$ $|8| \quad 8$

1.2 Integer Operations

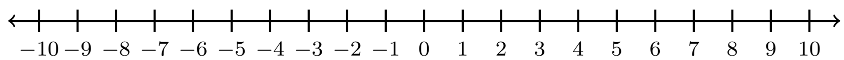
The ________ of a set of numbers is the result of their ___________.

The _____________________ is ________, because its sum with any other number is the other number. A positive number and its negative are each the _____________________ (or opposite) of the other because they sum to ________.

Example

Use the number line to evaluate the sum.

$-4 + 9$



Use tiles to evaluate the sum.

$8 + (-11)$

The _______________ of two numbers is the result of their _______________, which is the inverse of ___________. This means we can subtract a number by adding its ____________.

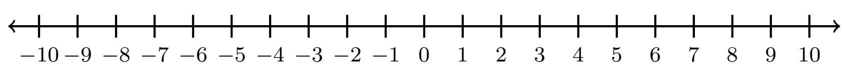
Example

Use tiles to evaluate the difference.

$5 - 7$

Use the number line to evaluate the difference.

$-3 - (-12)$



Write each difference as a sum. Then evaluate them.

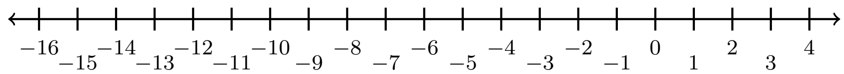
$6 - (-9)$ $\qquad$ $-8 - (-4)$ $\qquad$ $-5 - (-11)$

The ___________ of a set of numbers is the result of their _________________, which represents repeated ___________. For two factors, one factor ___________ how many times the other factor is __________.

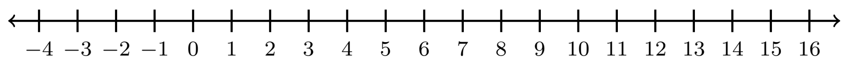
Example

Use the number line to evaluate each product.

$(-2) \cdot 7$



$(-3) \cdot (-4)$

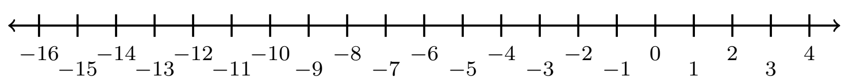


The ____________ of two numbers is the result of their ____________, which is the ____________ of multiplying. It asks what to multiply the ___________ (second number) by to get the ____________ (first number).

Example

Use the number line to evaluate each quotient.

$\dfrac{-15}{-3}$



$\dfrac{12}{-6}$

Notice that multiplying or dividing by a negative ______________ the sign (or direction) of the result. Therefore, the product or quotient of two ______________ numbers is ____________.

Example

Evaluate each product and quotient.

$5 \cdot 7$	$(-6) \cdot 9$	$8 \cdot (-4)$	$(-11) \cdot (-12)$
$\dfrac{56}{8}$	$\dfrac{-91}{7}$	$\dfrac{64}{-4}$	$\dfrac{-42}{-14}$

1.3 Rational Numbers

A ____________ is a number written as the ratio (quotient, division) of two numbers. It contains a _______________ on the top and a ________________ on the bottom.

A _____________________ is a number which can be written as a fraction using ____________ .

Example

Write each as a fraction to show that it is a rational number.

-19 $\qquad$ 2.8 $\qquad$ $0.\overline{3}$

In general:
- All _____________ are rational.
- All ___________________________ are rational.
- All _________________________ are rational.

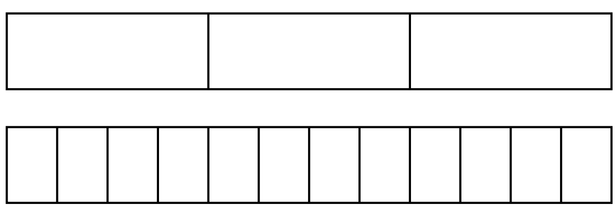
Fractions are ______________ if they represent the same number.

Example

Use the fraction bars to show that $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent.

$\frac{2}{3}$

$\frac{8}{12}$



A fraction can be _____________ by dividing both the numerator and denominator by their ________________________________.

Example

Simplify each of the following fractions.

$\frac{10}{35}$ $\qquad$ $\frac{20}{32}$

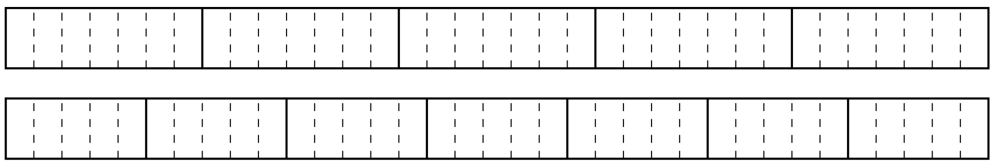
Fractions with different denominators are difficult to __________ and ______________, so its useful to write them with a _____________________________. The ____________________________________, which is the ______________________________ of the denominators, is preferred.

Example

Which is greater of $\frac{2}{5}$ and $\frac{3}{7}$?

$\frac{2}{5}$

$\frac{3}{7}$

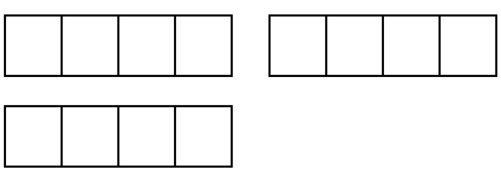


Write $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{2}{3}$ in ascending (least to greatest) order.

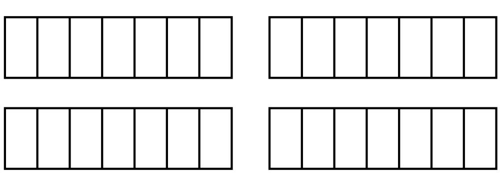
A ______________________ has a numerator ________ than the denominator, and is valued between zero and one. A fraction greater than one can be written as a _____________________, as the sum of an integer and a proper fraction; or as an _________________________, with a numerator _____________ than the denominator.

Example

Write the mixed number $2\frac{3}{4}$ as an improper fraction.



Write the improper fraction $\frac{25}{7}$ as an mixed number.



1.4 Adding and Subtracting Fractions

Fractions can be added or subtracted as long as they have a ____________________________, by adding or subtracting the ________________ and keeping the same _________________.

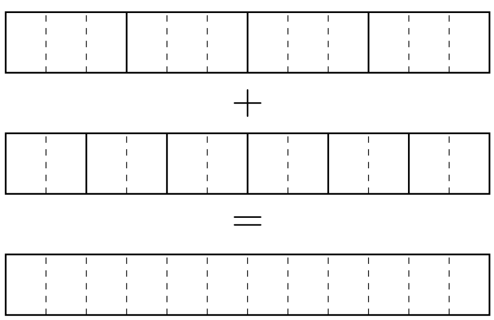
Example

Evaluate each of the following.

$\frac{5}{7} + \frac{4}{7}$ $\frac{1}{4} - \frac{3}{4}$ $\frac{1}{10} - \frac{7}{10} + \frac{9}{10}$

Example

Use the fraction bars to represent $\frac{3}{4}$ and $\frac{1}{6}$. Then find the sum of the fractions.



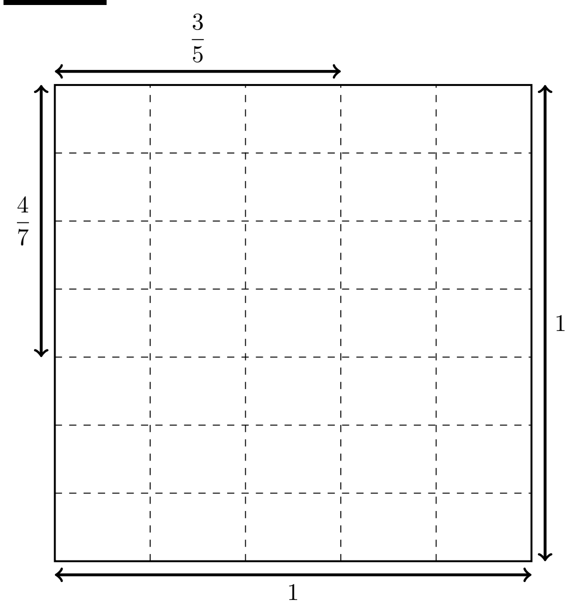
Example

Evaluate each of the following.

$\frac{9}{10} - \frac{18}{25}$ $\frac{2}{3} + \frac{4}{5}$ $2\frac{5}{8} - 4\frac{1}{4}$

1.5 Multiplying and Dividing Fractions

Example



Shade the region with dimensions $\frac{3}{5} \times \frac{4}{7}$.

How many equally sized sections make the 1 unit square?

How many equally sized sections are in the shaded region?

What fraction of the 1 unit square is shaded?

To multiply fractions, multiply the ______________ to get the resulting ______________, and multiply the ________________ to get the resulting ________________.

If multiplying an __________ by a fraction, write it as a fraction with ______ for the denominator.

If multiplying a _________________, write it as an _____________________ first.

Example

Evaluate each product.

$\frac{4}{9} \cdot \frac{3}{8}$ $\quad 2\frac{4}{5} \times \frac{1}{7}$ $\quad \left(-\frac{3}{10}\right)\left(\frac{20}{9}\right)$

The ________________________ is _______ because its product with any other number is the other number. The _____________ (or multiplicative inverse) of a number is another number which multiplies it to result in _______.

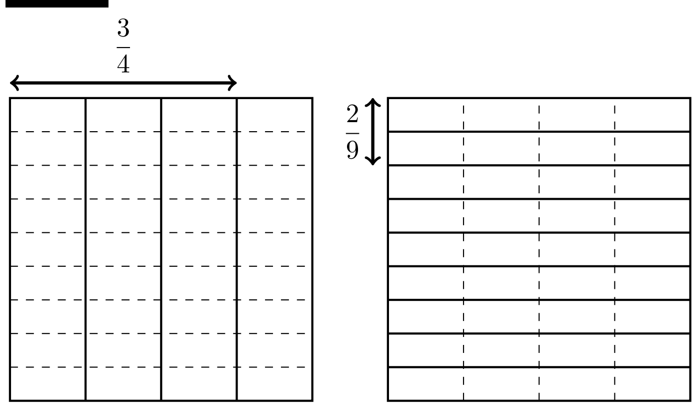
Example

Show that these numbers are reciprocals.

$\frac{5}{6}$ and $\frac{6}{5}$ $\qquad$ $\frac{1}{7}$ and 7 $\qquad$ $1\frac{1}{2}$ and $\frac{2}{3}$

The ______________ of a proper or improper fraction can be found by ______________ the numerator and denominator.

Example



Shade the regions showing $\frac{3}{4}$ and $\frac{2}{9}$.

How many small sections make $\frac{3}{4}$?

How many small sections make $\frac{2}{9}$?

How many times does $\frac{2}{9}$ fit into $\frac{3}{4}$?

____________ by a number is equivalent to ______________ by its ______________.

Example

Evaluate each quotient.

$\frac{5}{4} \div \frac{7}{8}$ $\qquad$ $\frac{3}{4} \div 6$ $\qquad$ $\frac{2}{3} \div \left(-\frac{6}{11}\right)$

$2\frac{1}{3} \div 3\frac{2}{5}$ $\qquad$ $9 \div \frac{3}{4}$ $\qquad$ $-2\frac{1}{5} \div (-3)$

1.6 Rational Number Equivalents

Decimals and Percents

“Percent” literally means to ________________, so 100% is equal to ___________.

- Convert percent to decimal: ________________.
- Convert decimal to percent: __________________.

Example

Convert the percentages to decimal numbers.

40% 83.1% 275%

Convert the decimal numbers to percentages.

0.7 0.042 4.2

Fractions to Decimals

All ____________________ can be written as an __________, a ______________________, or a _____________________. We can do this by treating a ___________ as __________.

Example

Write each fraction in decimal form without using a calculator.

$\frac{3}{5}$ $\frac{11}{25}$ $4\frac{3}{4}$

Write each fraction in decimal form using a calculator.

$\frac{97}{80}$ $\frac{8}{11}$ $\frac{49}{15}$

Decimals to Fractions

Each ________________ after the decimal point represents ___________ by a larger power of ten.

$0.1 =$ $0.01 =$ $0.001 =$ $0.0001 =$

Any __________________________ decimal can be written as a ____________. The number of _________ after the decimal point tells us how many ____________ the denominator should have.

Example

Write each as a fraction.

0.65 3.4 0.425 1.012

For _________________________, we can use the property that $0.\overline{9} = 0.99999... =$ _____.

$0.\overline{1} =$ $0.\overline{1} =$ $0.\overline{1} =$

Example

Write each as a fraction.

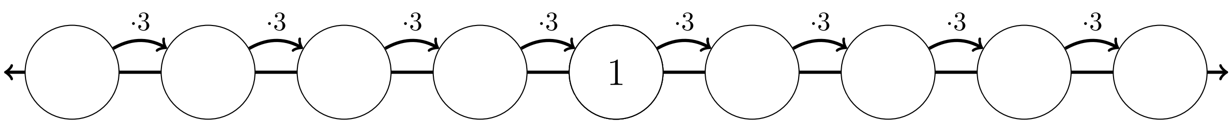
$0.\overline{6}$ $0.\overline{45}$ $0.\overline{259}$

$0.7\overline{3}$ $0.11\overline{8}$ $0.1\overline{28}$

2.1 Positive and Negative Exponents

An expression in the form ________ can be used to represent repeated __________________. The _________, a, is the value to be multiplied, and the ______________, m, is the number of a's being multiplied. We can read the expression as "a to the ___________ of m".

Here are some of the powers when the base is 3:



Example

Write the expressions in expanded form, and then evaluate them.

3^4 4^3 11^2

Write the expressions in expanded form.

x^6 y^5 a^4

Write the expressions in exponent form.

$7 \cdot 7 \cdot 7$ $12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$ $x \cdot x$

If the exponent is ______________, we need to repeat the ______________ of multiplication, which is ____________. If the base is an integer, this usually results in a _____________.

Example

Write the expressions in expanded form, and then evaluate them.

3^{-2} 2^{-5} 10^{-3}

Write the expressions in expanded form.

x^{-4} y^{-2} b^{-7}

Write the expressions in exponent form.

$\frac{1}{6 \cdot 6 \cdot 6}$ $\frac{1}{9 \cdot 9 \cdot 9 \cdot 9}$ $\frac{1}{y \cdot y \cdot y \cdot y \cdot y \cdot y}$

2.2 Exponent Rules with the Same Base

Example

Write these expressions in expanded form, then simplify as single exponents.

$3^5 \cdot 3^2$

$\frac{5^9}{5^3}$

Rule 1: The Exponent Product Rule

Multiplying expressions with the **same base** is equivalent to ______________________________.

Rule 2: The Exponent Quotient Rule

Dividing expressions with the **same base** is equivalent to ____________________________________.

Example

Simplify each using the Exponent Product Rule.

$2^8 \cdot 2^3$ $7^6 \cdot 7^{13}$ $x^5 \cdot x^9$

Simplify each using the Exponent Quotient Rule.

$\frac{6^{14}}{6^5}$ $\frac{4^3}{4^8}$ $\frac{t^{10}}{t^7}$

Example

Write these expressions in expanded form, then simplify using single **positive** exponents.

$\left(2^3\right)^4$

a^{-5}

Rule 3: The Exponent Power Rule

Raising a base to a power then another is equivalent to __________________________________.

Rule 4: The Negative Exponent Rule

Changing the sign of an exponent is equivalent to taking the ______________ of the expression.

Example

Simplify each using the Exponent Power Rule.

$(3^4)^2$ $(10^5)^3$ $(b^7)^6$

Write using a positive exponent.

5^{-7}

Write without using a fraction.

$\dfrac{1}{e^{11}}$

Special Exponent Values

Any exponential expression with zero for the exponent (and the base is not zero) ______________________.

Any exponential expression with one for the exponent ____________________________.

Example

Simplify each expression with a positive exponent. State which rule is used in each step.

$\dfrac{t^8}{t^{11}} \cdot t^5$

$s^5 \left(s^4\right)^7$

$\dfrac{(a^2)^3}{a^{13}}$

$\dfrac{b^{22}}{(b^2 \cdot b^4)^3}$

Example

Simplify each expression.

$a^3b^5 \cdot a^7b$

$\dfrac{x^5y^2}{x^4y^8}$

$\dfrac{s^4t^5 \cdot s^2}{t^2}$

2.3 Exponent Rules with the Same Exponent

Example

Write these expressions in expanded form, then simplify each using a single base.

$2^4 \cdot 3^4$

$\dfrac{12^5}{4^5}$

Rule 5: The Base Product Rule

Multiplying expressions with the **same exponent** is equivalent to ____________________________.

Rule 6: The Base Quotient Rule

Dividing expressions with the **same exponent** is equivalent to __________________________.

Example

Simplify each of the following. Write your answer as a single exponent.

$3^7 \cdot 5^7$

$2^4 \cdot 9^4$

$\dfrac{63^5}{9^5}$

Simplify and evaluate each of the following.

$\dfrac{(2^5 \cdot 3)^3}{2^{11} \cdot 3^2}$

$\dfrac{10^2 \cdot 10^4 \cdot 5}{5^7}$

Simplify each of the following. Don't use fractions for your final expressions.

$\dfrac{(ab)^2}{b^5}$

$\dfrac{(3x)^4}{x^5}$

2.4 Scientific Notation

The ____________ number system is base_______, which means each ________________ corresponds to a different power of ten.

- If n is ___________, then 10^n is 1 shifted n place values to the _______.
- If n is _____________, then 10^n is 1 shifted $|n|$ place values to the _________.

Example

Write in decimal notation:

10^5 10^{-4} 10^3

Write as an exponent of 10:

0.000001 10 000 000 0.01

________________________ is a way of writing numbers which uses __________________ multiplied by a ____________________. The leading digits always have a ______________________________ before the decimal point, with the power of ten used to shift the _________________.

Scientific notation with ____________ powers can represent _____________ numbers, and scientific notation with _____________ powers can represent ________________ numbers.

Example

Write in ordinary decimal notation:

7.482×10^5 5.213×10^{-4} 3.9742×10^3

Write in scientific notation:

0.00000358 34 910 000 0.0882

These are not in valid scientific notation. Correct them.

12.3×10^8 0.0234×10^5

The exponent on the ten is sometimes called the ___________________________. To compare two numbers in scientific notation, compare the ___________________________ first. If these are the same, the numbers have similar size, so we compare their __________________.

Example

Which is larger of 7.452×10^{-6} and 3.529×10^{-2}?

Compare the sizes of a bacterium with a diameter of 1.5×10^{-6} m, a virus with a diameter of 4.5×10^{-8} m, and a red blood cell with a diameter of 8.2×10^{-6} m.

2.5 Operations in Scientific Notation

To ____________ and __________ numbers in scientific notation, the ___________________ can be treated as ordinary numbers, and the _______________ can be simplified using exponent rules. Always check that the answer is in correct ________________________.

Example

Evaluate each of the following.

$\left(3.5 \times 10^{8}\right)\left(5 \times 10^{-3}\right)$

$\dfrac{1.8 \times 10^{11}}{6 \times 10^{7}}$

$\left(5 \times 10^{-4}\right)\left(9 \times 10^{-9}\right)$

$\dfrac{5.6 \times 10^{5}}{8 \times 10^{18}}$

Example

The earth is 1.496×10^{11} m from the sun. Light travels at 3.0×10^{8} m each second. How many seconds does it take light from the sun to reach the earth? *Use a calculator.*

2.6 Square Roots

If we want to make a square whose sides are _____ units long, we'll need ________________ unit squares. This is why multiplying a number by _________, or applying an exponent of _______ is called _____________.

6 units

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

6 units

Example

How many unit squares form a square with sides six units long?

The _____________ (the opposite) operation of squaring is the _________________, which is represented by the __________ symbol $\sqrt{\ }$. The number underneath a radical is called the _____________.

________ is the number whose square is equal to _____.

Example

What is the side length of a square made from 36 unit squares?

A number which results from squaring a whole number is called a ____________________:

$1^2 =$	$5^2 =$	$9^2 =$	$13^2 =$	$17^2 =$
$2^2 =$	$6^2 =$	$10^2 =$	$14^2 =$	$18^2 =$
$3^2 =$	$7^2 =$	$11^2 =$	$15^2 =$	$19^2 =$
$4^2 =$	$8^2 =$	$12^2 =$	$16^2 =$	$20^2 =$

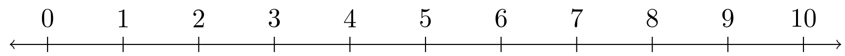
The _________________ of a perfect square is a ____________________. The square root of any other whole number is ______________ whole numbers. These square roots can only be __________________ when using finite decimal places.

Example

Evaluate $\sqrt{289}$, and give a reason for your answer.

Example

Approximately locate $\sqrt{52}$ on a number line. Explain why the estimate has this location.



Approximate the value of $\sqrt{25}$ with a calculator.

2.7 Understanding Irrational Numbers

A _______ is a collection of mathematical items, which is often a collection of _____________.

- The _____________________ are the numbers used for counting, including_________.
- The ____________ are the whole numbers along with their _____________ counterparts.
- The ______________________ are the numbers which can be written as a ___________ (or "ratio") with two integers.

Two new number sets to consider:

- The ___________________ are the numbers which can be placed on the _________________.
- The ________________________ are the _______ numbers which are not ___________.

Rational and Irrational Numbers

We've already seen that ____________, ________________ decimals and ______________decimals can all be written as fractions using integers, so they are ________________________________. In fact, ______________________________ is one of these three.

Therefore, any other number must be an _______________________.

A decimal which ________________________ and ____________________ is _____________.

The ________________ of a whole number which is not a perfect square is _____________.

$\pi = 3.14159\ldots$ is _____________.

Combining Rational and Irrational Numbers

The sum or product of two rational numbers is __________ ___________.

Why this is true:

If two numbers are ___________, that means they can be represented by _____________. Adding two fractions makes a ___________, and multiplying two fractions makes a ___________, so the ________ or ___________ is ____________________.

Another way of describing this is to say that the rational numbers are __________ under addition and multiplication. Just like you can't leave a room if it is __________, we can't leave the closed ______________________ by adding or multiplying.

The sum or product of two irrational numbers is _______________ irrational, but not ___________.

Example

Think of a pair of irrational numbers whose sum is rational.

Think of a pair of irrational numbers whose product is rational.

This means the irrational numbers are _______________ under addition or multiplication.

The sum of a rational and irrational number is ___________ _____________.

The product of a (non-zero) rational number and an irrational number is ___________ _____________.

Example

Answer true or false. Give a reason for each answer.

The product of a rational number and an irrational number is never irrational.

$3 + \pi$ is a rational number.

$\frac{2}{3} \cdot \sqrt{25}$ is irrational, because it is a product of a non-zero rational number and a square root.

3.1 The Order of Operations

A ____________________________ is a combination of _____________ and _______________ which represents a numerical _________. To _____________ an expression means to determine that overall value. When evaluating expressions, we follow the ___________________________.

G	_________________________ including: (**in parentheses**), [**in brackets**], {**in braces**}, |**in absolute value bars**|, √**under a radical**, and **numerator of a fraction** / **denominator of a fraction**.
E	________________, which includes **evaluating**$^{\text{powers}}$ and √**evaluating radicals**.
M D	__________________ and _____________, in order from **left-to-right**.
A S	_____________ and _________________, in order from **left-to-right**.

To show your working clearly, you should write your calculations ____________________________.

We use the __________ symbol to indicate that expressions as equivalent. You should always work _____________, with all the equals signs written in a _________________.

Example

Evaluate each expression.

$3\,(8-3)^2 - 5 \cdot 7$

$\dfrac{4-3(-6)}{5(-3)+17}$

Evaluating Exponents

Example

Write each expression in expanded form, and then evaluate.

$(-2)^3$

$(-2)^4$

-2^3

-2^4

- A negative base to an _______________ is always _____________.
- A negative base to an _________________ is always ____________.
- A negative sign not contained in _______________________ with the base is not part of the base, and will be evaluated _________ the exponent.

Example

Evaluate each of the expressions.

$(-3)^4 + (-4)^3$

$(-3)^2 + (-3)^3 - 3^4$

Expressions Represented with Words

related to +	related to −	related to ×	related to ÷
plus	minus	times	divide
sum	difference	product	quotient
addition	subtraction	multiplication	division
more than	less than	twice, double, triple	half of, third of
increased by	decreased by	of	split evenly

Example

Write each description as a numerical expression, then evaluate.

The quotient of 20 and 4.

25 less than 8.

Twice the difference of 13 and 9.

10 more than the product of 9 and 7.

Half of the sum of 14 and 8.

The sum of 14 and half of 8

7 subtracted from the square root of 16.

The square of the quantity 18 minus 7.

3.2 Variables and Substitution

A ____________ is a quantity whose value we ________________________ or whose value can ____________. A variable is usually represented by a __________.

An ______________________________ is an expression which contains ______________ as well as numbers and operations.

If we know the values of the variables, we can _______________ the variables by replacing them with their values. This turns an ____________________________ into a numerical expression, which can be ______________. Always surround values with __________________ when substituting.

Example

Suppose that $a = 5$, $b = -7$, and $c = 2$. Evaluate each expression using these values.

$2a + 3b$

$\sqrt{b^2 - 4ac}$

Example

Write each as an algebraic expression, where value of "a number" is represented by n.

Triple the sum of a number and 5.

10 less than the square of a number.

Evaluate each expression where the value of "a number" is 2.

Evaluate each expression where the value of "a number" is -8.

Example

Penelope's Perfect Pizza sells large pizzas for \$6 each, and also charges \$8 for delivery.

Choose a variable to represent the number of pizzas delivered to a customer.

Write an expression representing the total cost to a customer.

Use your expression to find the cost to a customer who orders 4 pizzas.

Parts of an Algebraic Expression

___________ are the parts of an expression separated by ________ and __________ symbols. A term is often written as a ____________ of a number and variables, sometimes with _______________.

The _______________ of a term is the ____________ which multiplies the _____________ in the term. The ________ of the coefficient is determined by the operation ___________ the term.

A ____________________ is a term which doesn't contain any variables.

Example

List the terms of the expression $2x^2 + 3xy - 7y^2 + x - 9y + 14$.

What are the coefficients of the terms?

The coefficient of x^2 is

The coefficient of y^2 is

The coefficient of xy is

The coefficient of x is

The coefficient of y is

What is the constant term?

3.3 Combining Like Terms

Two expressions are ______________ if their values are _______________ as each other for any values of their _____________ .

Example

Complete the tables by evaluating the expressions.

x	$7x$	$2x$	$7x+2x$	$9x$
-3				
-1				
2				
5				

What do you notice?

What do you wonder?

x	$3x$	$3x+8$	$11x$
-2			
1			
4			
6			

What do you notice?

What do you wonder?

x	y	$6x$	$4y$	$6x+4y$	$10xy$
-2	3				
1	5				
4	-1				
6	7				

What do you notice?

What do you wonder?

________________ are two or more terms whose combinations of _____________ are _______________.

Constant terms are also considered to be _______________ with each other.

Expressions with like terms can be ______________ by _____________________________ into an ______________ single term by adding the ________________.

Example

Does $7x+2x$ have like terms? Does $3x+8$ have like terms? Does $6x+4y$ have like terms?

Example

If these are like terms, simplify them. If they are not, explain why.

$6a + 10a$

$4s - 9t$

$5y^2 - 12y^2$

$-2n^2 + 5n$

$-3 + 8$

Example

Simplify $4x + 5x - 8y + 6y + 7 - 3$.

COMMUTATIVE PROPERTY OF ADDITION

Sums with ____________ terms are ____________.

COMMUTATIVE PROPERTY OF MULTIPLICATION

Products with ____________ factors are ____________.

Example

Simplify each of the following expressions by combining like terms.

$5s + 4t - 8s + 6t$

$4x - 15x - 9 + 7x$

$9cd - 2dc$

$7ab - 6a + 3b + 5ba$

$3x^2y + 2yx^2 + 9xy^2$

$5x + 7x^2 - x + x^2$

3.4 The Distributive Property

Example

Complete the table by evaluating the expressions.

x	$x+4$	$3(x+4)$	$3x$	$3x+12$
-3				
1				
5				
10				

What do you notice?

What do you wonder?

THE DISTRIBUTIVE PROPERTY

Multiplying a sum by a value is _______________ to multiplying each term of the sum by that value before adding.

The process of applying the distributive property is called ________________. The ________________ helps us to make sure that each term __________ the parentheses is multiplied by the value ___________ the parentheses.

Example

Distribute each of the expressions.

$5(x+9)$ $-2(y-7)$ $7(2n-3)$

$t(t+7)$ $-3p(q+5)$ $2u(3u-5)$

$-4(3a-5b-9)$ $2x(x+3y-5)$

3.5 Factoring

______________ is the opposite process of ________________. One way to do this is to find the ________________________________, or _________.

The first factor to find is the ________________________________ of all the _________________.

Example

Factor the following expressions.

$7n - 21$	$10x + 16$	$15m - 50$
$6a - 30$	$28x + 70$	$105t + 45$

If all the __________ share any ______________ in common, these are also factors of the GCF.

Example

Factor the following expressions.

$x^2 + 8x$	$y^2 - 12y$	$2a^2 - 14a$
$8st + 4t$	$12x^3 + 15x^2$	$4a^2b - 7ab$

3.6 Algebraic Reasoning

Much of what we do in ___________ is based on the following __________________________.

associative property of addition		if we add three numbers, we can do either addition first
associative property of multiplication		if we multiply three numbers, we can do either multiplication first
commutative property of addition		we can change the order of terms in addition
commutative property of multiplication		we can change the order of factors in multiplication
distributive property		we can distribute and factor

Most of the time we don't need to think about these properties to work algebraically. Sometimes, however, we need to __________ our work by _____________ our reasoning, using the properties.

We can also use _______________ as reasons for our calculations.

Example

Justify the following simplification, giving a reason for each step.

$$3(x-4)+2(5x+7) = 3(x)+3(-4)+2(5x)+2(7)$$

$$= 3(x)+3(-4)+(2\cdot 5)x+2(7)$$

$$= 3x+(-12)+10x+14$$

$$= 3x+10x+(-12)+14$$

$$= (3+10)\cdot x+(-12)+14$$

$$= (3+10)\cdot x+(-12+14)$$

$$= 13x+2$$

4.1 Solving Equations

An ____________ is a mathematical statement which says that two ________________ are __________.

If the equation contains a ____________, the value of that ____________ which makes the equation ________ (makes the two sides __________) is called a ____________.

Example

Consider the equation $\frac{3x+6}{5} = -3$.

Show that $x = -7$ is a solution.

Show that $x = 8$ is **not** a solution.

____________ an equation means to ____________________ for it.

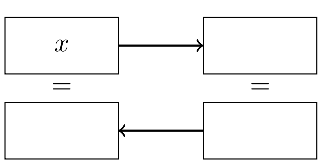
Solving Method 1: Backtracking

The ________________ method identifies the ______________ applied to the variable, then uses ________________________ to work back to the ____________.

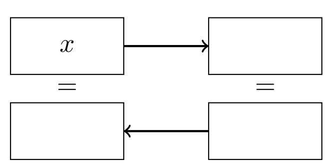
Example

Solve each equation using the backtracking diagram.

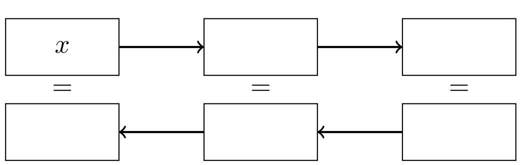
$x + 11 = 7$



$6y = 18$

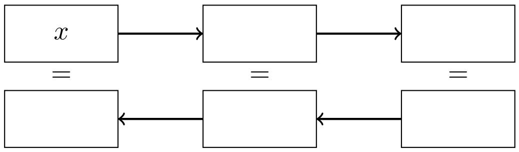


$-5(t - 8) = 30$

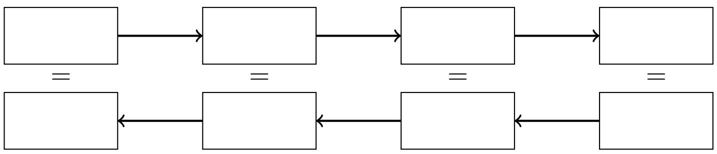


Example (continued)

$\frac{n}{4} + 11 = 5$



$\dfrac{3x-9}{2} = 6$



The Properties of Equality

addition property of equality	$a = b$ if and only if $a + c = b + c$
subtraction property of equality	$a = b$ if and only if $a - c = b - c$
multiplication property of equality	$a = b$ if and only if $a \cdot c = b \cdot c$ (if $c \neq 0$)
division property of equality	$a = b$ if and only if $\frac{a}{c} = \frac{b}{c}$ (if $c \neq 0$)

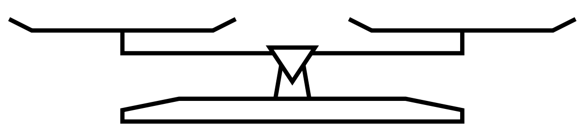
Solving Method 2: Balancing Each Side

We can imagine an equation as a ________ whose two sides perfectly ____________. The scale remains ______________ as long as we always do the ___________________________.

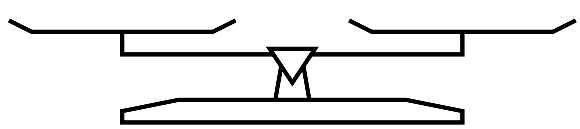
Example

Use the balance scales to illustrate each equation as you solve them.

$x + 5 = 12$

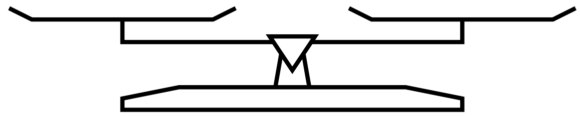


$4x = 20$



Example (continued)

$5x + 9 = 24$



Example

Solve each equation.

$n - 17 = -3$

$\frac{b}{7} = 9$

$-3t = -39$

$2u - 9 = 15$

$\frac{x + 15}{4} = 3$

$2(y + 5) - 7 = 27$

Example

Jessica is a member of a gym that charges \$45 for membership, and an extra \$6 for each visit. Jessica has paid \$87 in total to the gym. How many visits has Jessica made to the gym?

Choose and define the variable.

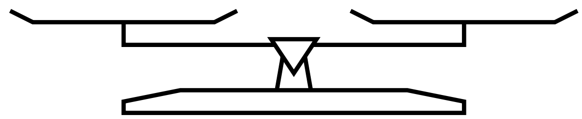
Solve the equation.

Write the problem as an equation.

4.2 Equations with Simplifying

Example

Use the scale to illustrate $3x + 5 + 2x + 7 = 27$, and solve it.

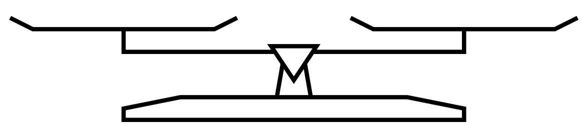


Solve $6t - 9 - 8t + 21 = 2$.

Solve $7 - 8n + 5n + 12n = 65 + 32$.

Example

Use the scale to illustrate $7x + 2 = 4x + 8$, and solve it.

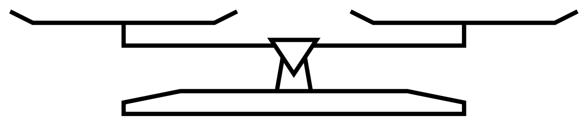


Solve $5a = 56 - 2a$.

Solve $17 - b = 35 + 2b$.

Example

Use the scale to illustrate $2(3x+2) = x+9$, and solve it.



Solve $9k + 16 - 6(k+8) = 10$.

Solve $2(y-5) + 3(4y+7) = -17$.

Solve $3\,(w+2) = 2\,(w-5)$.

Solve $7(z-9) = -5(z+3)$.

1. If there are any **parentheses**, ______________ them.
2. If the variable is on **both sides**, remove the term from one side by ____________________________.
3. If the variable is **repeated on one side**, simplify by ____________________________.
4. Finish solving as using _________________________.

Example

Jayden starts jogging at a speed of 2 meters per second. Hailey waits 90 seconds, then starts jogging at a speed of 2.5 meters per second. How long will it take for Hailey to pass Jayden?

Choose and define the variable.

Solve the equation.

Write the problem as an equation.

4.3 Equations with Fractions

Approach 1: Solve while keeping fractions

When solving equations with ____________, we can still ____________ them and use ________________________ to solve them as we would for equations with integers only.

Example

Solve $\frac{2a}{3} + \frac{5}{6} = \frac{4}{3}$.

Solve $\frac{2t+11}{4} + \frac{5t}{8} = \frac{16}{5}$.

Approach 2: Eliminate denominators first

Example

For each list of fractions, find the lowest common denominator.

$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ $\frac{2}{3}, \frac{1}{5}, \frac{7}{10}$ $\frac{5}{4}, \frac{1}{6}, \frac{11}{12}$

Multiply each fraction by the lowest common denominator, and simplify.

What do you notice? What do you wonder?

The denominator of a fraction can be eliminated by________________ the fraction by a____________ of the denominator. The _________ is a multiple of all the denominators in a set of fractions. This means we can eliminate all denominators in an equation by ______________________________ by the _________ of all the fractions in the equation.

Example

Eliminate the denominators first before solving the equations.

Solve $\frac{2a}{3} + \frac{5}{6} = \frac{4}{3}$. Solve $\frac{2t+11}{4} + \frac{5t}{8} = \frac{16}{5}$.

Which of the two approaches did you prefer? Why?

4.4 Number of Solutions

A ____________ to an equation is a value for the ____________ which makes the equation _________. Many equations have ___________________________, but this is not always the case.

Example

Use the table analyze the equation $3x + 5 = 3x + 7$.

x	LHS $3x + 5$	RHS $3x + 7$	Solution? LHS $\stackrel{?}{=}$ RHS
-2			
1			
4			
9			

What do you notice?

What do you wonder?

Use the table analyze the equation $2(x - 3) = 2x - 6$.

x	LHS $2(x-3)$	RHS $2x - 6$	Solution? LHS $\stackrel{?}{=}$ RHS
-2			
1			
4			
9			

What do you notice?

What do you wonder?

If the two sides of an equation differ by a _____________, then _________________ is a solution.

If the two sides of an equation are ______________, then ____________________ is a solution.

NUMBER OF SOLUTIONS
for linear equations with both sides distributed and simplified

variable term	constant term	type of solution
same coefficient	different constants	
same coefficient	same constant	
different coefficients	N/A	

Example

Determine the number of solutions each equation has. Justify your answers.

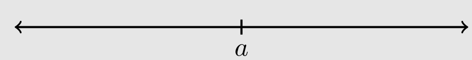
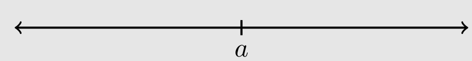
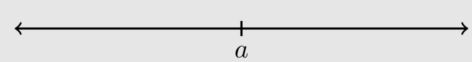
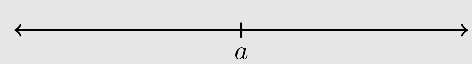
$3(2x+4)-2x+8=4(x+5)$

$4x+3-2(x-1)=5x+8$

$2(5x-3)+4x=7(2x-1)$

4.5 Linear Inequalities

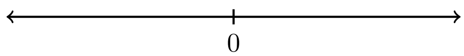
An ____________ is a statement similar to an ___________, but doesn't use _________.

x **is less than (not equal to)** a		
x **is greater than (not equal to)** a		
x **is less than or equal to** a		
x **is greater than or equal to** a		

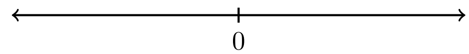
Example

Write each description as an inequality, and plot it on the number line.

x is below 3.



x is at least -7.

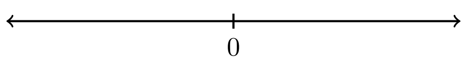


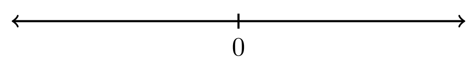
Example

Use the tables analyze the inequalities $2x - 7 > 9$ and $-3t + 5 \geq -1$. Then plot the solution on the number line.

x	$2x - 7$	$2x - 7 \overset{?}{>} 10$
5		
8		
10		
12		

t	$-3t + 5$	$-3t + 5 \overset{?}{\geq} -1$
-1		
1		
2		
5		





What do you notice?

What do you wonder?

Solve the inequalities algebraically.

When solving inequalities, apply the same operation to ________________.

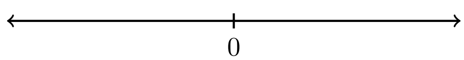
To add or subtract	To multiply or divide a positive	To multiply or divide a negative

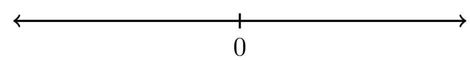
Example

Solve each inequality, and use a number line to represent the solution set.

$3y - 5 > 10.$

$-8u + 12 \leq -4.$





Example

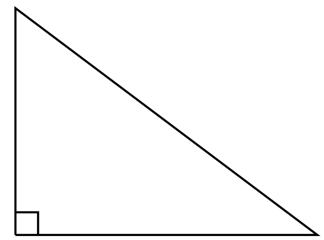
Ben can save \$180 each week, but he currently owes the bank \$630. He can afford to go on vacation once he has more than \$4500 saved in his bank account. When can Ben afford to go on vacation?

Choose and define the variable.

Solve the inequality.

Write the problem as an inequality.

5.1 The Pythagorean Theorem



A ________________ is an angle which measures _______ .

A __________________ is a triangle with a ________________.

The ___________ side of a right triangle is called the ________________. The other two sides are called the ________.

Notice that the ________ are ____________ to the right angle (they touch it), while the ________________ is not.

THE PYTHAGOREAN THEOREM

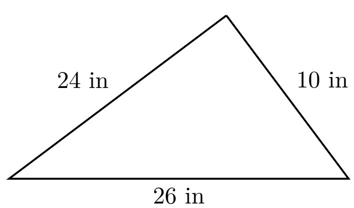
Let a, b and c be the lengths of the sides of a triangle, where c is the longest side.

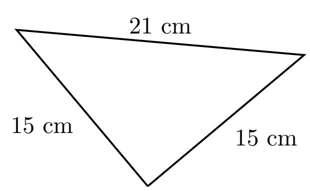
The triangle is a __________________ if and only if

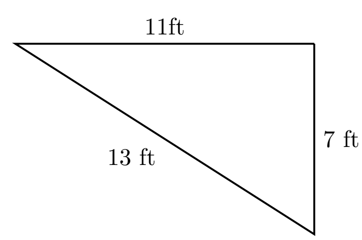
This means that in a right triangle a and b are the lengths of the ________, and c is the length of the ________________ . It's always a good idea to ________ the sides a, b and c when working a right triangle problem.

Example

Determine if the following triangles are right triangles.







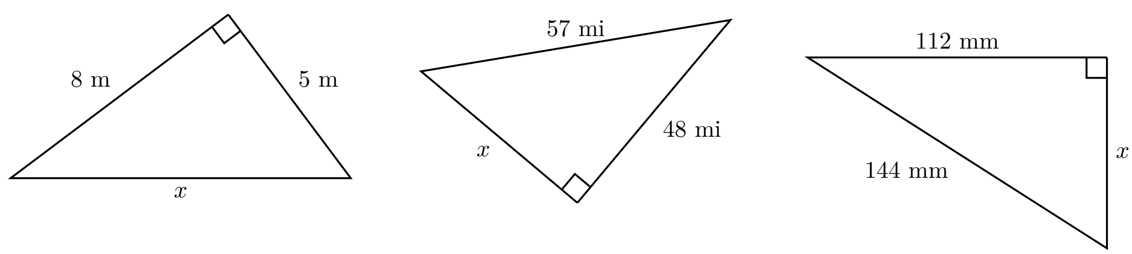
5.2 Lengths in Right Triangles

If we know that a triangle is a ___________________, and we know the lengths of ______________, we can find the length of the ________________ using the ____________________________.

Don't forget that _____ is always assigned to the length of the ________________, and that _____ and _____ are assigned to the ________. Always check that the ________________ works out to be the ____________ side.

Example

Find the length x in each triangle.



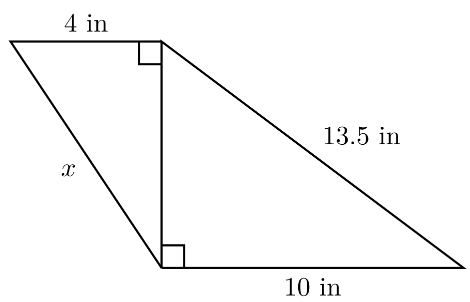
Example

A ship sails 100 miles north from its dock, then turns east and sails 150 miles. How far is the ship from the dock?

5.3 Multi-Step Right Triangle Problems

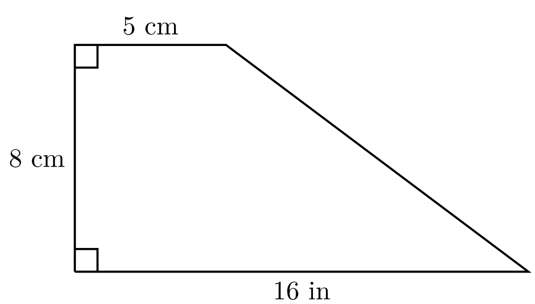
Example

Find the length x.



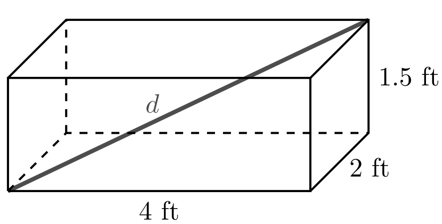
Example

Find the perimeter of the trapezoid.



Example

Find the length of the diagonal d.



5.4 Distances on the Coordinate Plane

Coordinate Plane Review

The ______________________ represents the values of two ______________ with a _________. Its ______________ position is value of _____, and its ____________ position is the value _____.

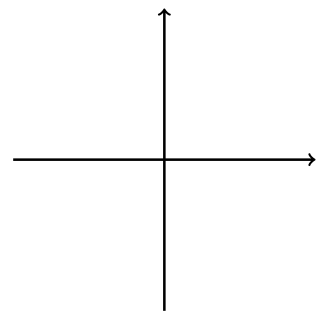
An _________________ is written as __________. It too represents the values of the ______________ x and y, in that order, and the ________________ of a point on the plane.

The __________ is the horizontal line where ___________.

The __________ is the vertical line where ___________.

The _________ is where the axes intersect, at the point _________ .

The ______________ are the four regions separated by the axes.



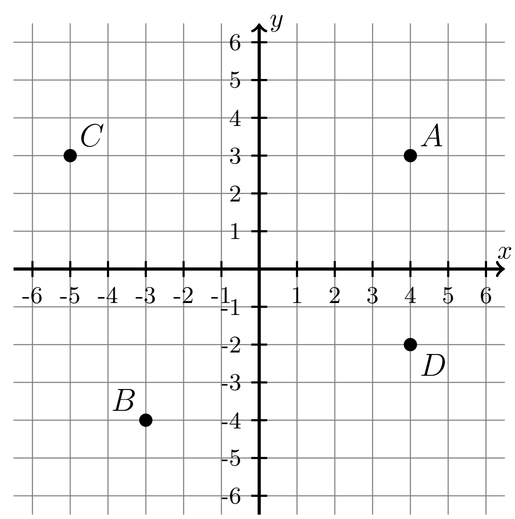
Example

a) Write down the coordinates of A, B and C.

b) What variable values does D represent?

c) Plot and label the points $(-4, 1)$, $(4, -3)$ and $(-3, -5)$.

d) Plot and label the point representing $x = 2$ and $y = 6$.



Calculating Distances

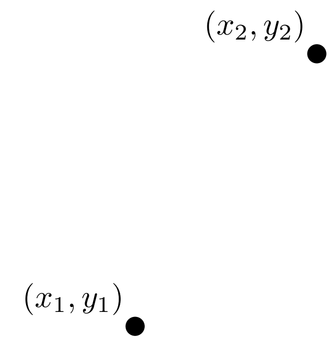
Example

Consider the distances between A, B, C and D above. What are the two simplest distances to find?

Why are these distances simpler to find than the others?

The __________ between two points is the same as the _________ of a line segment between them. We can form a ________________ with the distance as the ______________ and horizontal and vertical line segments as _______ .

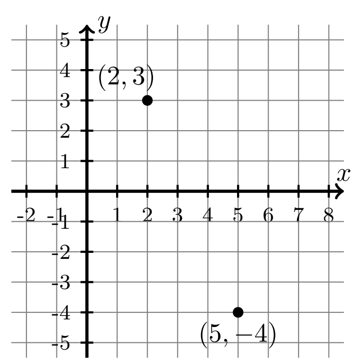
The lengths of these _______ represent the ___________ in x and y between the two points. The Greek letter ________, Δ can be used to mean the _____________ a variable.



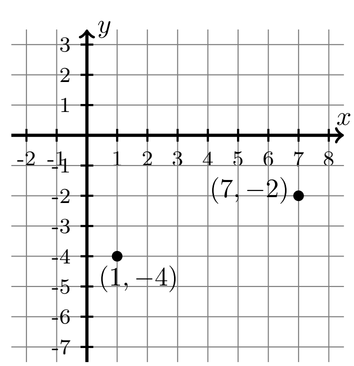
THE PYTHAGOREAN THEOREM for the distance between points d

Example

Find the distance between $(2, 3)$ and $(5, -4)$.



Find the distance between $(1, -4)$ and $(7, -2)$.



6.1 Function Rules and Tables

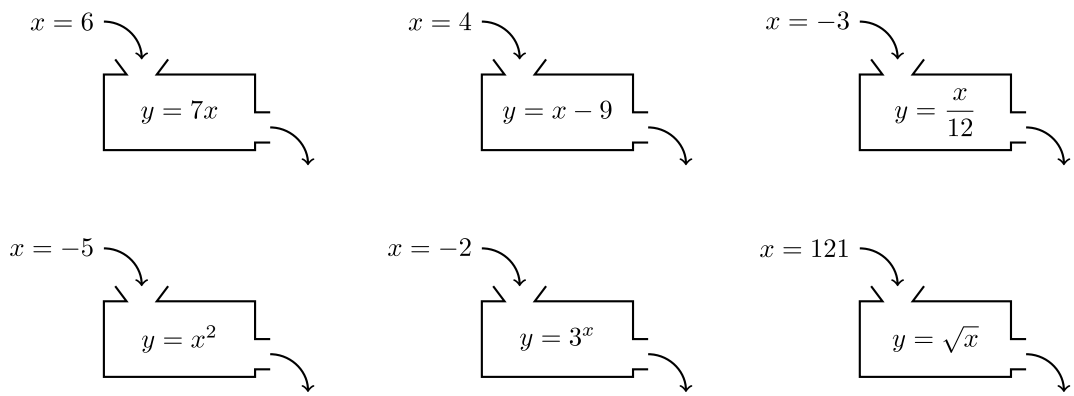
A __________ is a collection of ordered pairs which represents a relationship between two __________.

A __________ is a relation where the value of the ______________________, usually x, determines the value of the ____________________, usually y. Each ________ (x value) in a function produces exactly one ________ (y value).

Two ways to represent functions are ________________ and ________.

Example

Write in the output for the following function machines.



Example

For the function with the rule $y = 2x + 5$, determine the output for each input.

$x = 3$ $x = -6$ $x = 7.5$

For the function with the rule $y = x^2 - 9$, determine the output for each input.

$x = 1$ $x = -3$ $x = 4.5$

Example

Complete the table for the function $y = 4x - 11$.

input x	-3	-2	-1	0	1	2	3	4	5
output y									

Complete the table for the function $y = -3x + 5$.

input x	-6	-4	-3	0	1	2	5	7	10
output y									

Complete the table for the function $y = x^3$.

input x	-4	-3	-2	-1	0	1	2	3	4
output y									

6.2 Finding Linear Rules from Tables

A ____________________ is a function whose output results from ______________ the input by a constant and __________ another constant. All ____________________ can be written in the same __________________.

LINEAR FUNCTION GENERAL FORM

where m and b are constant.

Example

Find the constants m and b for these linear functions.

$y = -3x + 7$ $y = \frac{x}{4} - 9$ $y = 9x$

$y = 13 - 7x$ $y = -4(x - 5)$ $y = \frac{3x+4}{6}$

The ____________________ between two points of a function is the ________ of the ___________ in the __________ and the ____________ in the ________.

Example

Complete the table for each function. Then find the rate of change between each pair of points.

$y = 4x - 2$

input x	output y
0	
1	
2	
3	
4	

$y = -\frac{2}{3}x + 5$

input x	output y
-3	
0	
3	
6	
9	

What do you notice? What do you wonder?

THE RATE OF CHANGE OF A LINEAR FUNCTION

Linear functions are functions with a ________________________________,

which is the ______________________ in the general form.

Example

Find a rule for the linear function described in each table.

input x	output y
0	7
1	12
2	17
3	22

input x	output y
-4	5
-1	-1
1	-5
5	-13

input x	output y
0	8
8	14
12	17
24	26

To find a rule from linear table:

Step 1. Use the table to calculate the _____________________. This is ______.

Step 2. Using m and the values for x and y from one point, ________ for ____.

Step 3. Use m and b to __________________________.

Step 4. Check that the rule is ________ for the values in the table.

6.3 Plotting Function Graphs

An __________________, written as __________ has two equivalent meanings:

- The values of the two _____________, x and y, in that order.
- The ________________ of a point on the coordinate plane.

A ____________ describes a relationship between values of x and values of y. This means we can represent a ____________ by plotting a __________ on the coordinate plane.

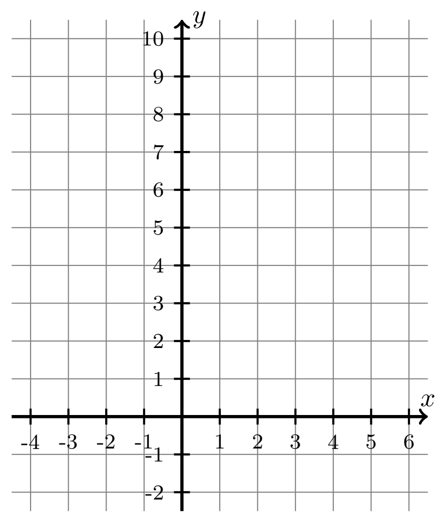
Example

Complete the table for the function $y = 2x + 4$.

x	-3	-2	-1	0	1	2	3
y							

Write the entries from the table as a list of ordered pairs.

Plot a graph of the function on the coordinate plane.

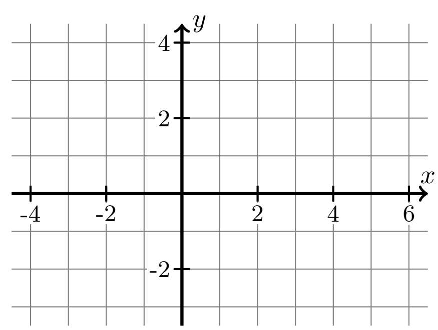


Example

Complete the table for the function $y = \frac{x}{4} + 1$.

x	-4	-2	0	2	4	6
y						

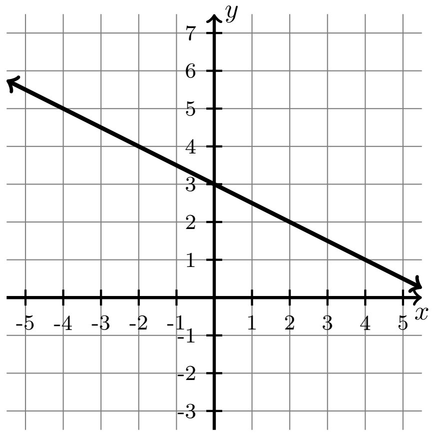
Plot a graph of the function on the coordinate plane.



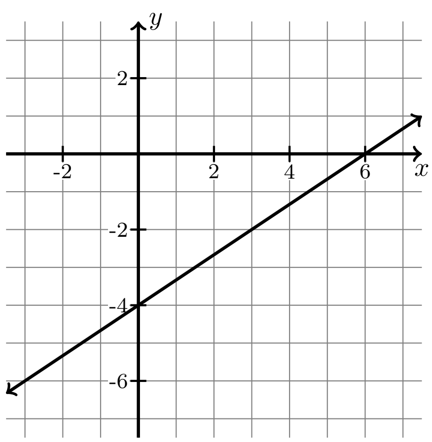
A function of the form $y = mx + b$ is called a ____________________ because its graph is a _________________.

Example

Complete the tables and find the rules for the functions shown in the graphs.



x	y



x	y

6.4 Identifying Linear and Nonlinear Functions

A _________________________ is a function which is not a _____________________.

LINEAR FUNCTIONS vs. NONLINEAR FUNCTIONS

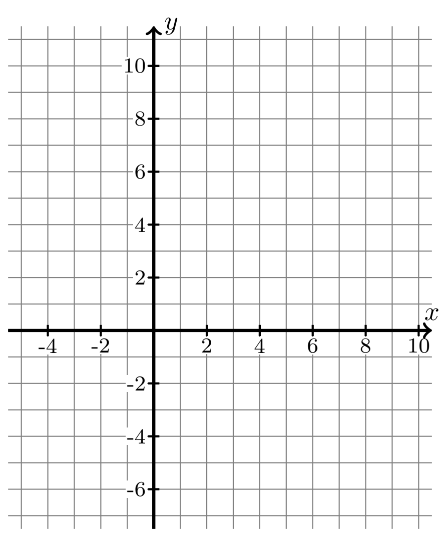
	linear functions	nonlinear functions
rule	can be written as $y = mx + b$	can't be written as $y = mx + b$
table	constant rate of change	changing rate of change
plot	straight line	not a straight line

Example

Does the rule $y = -\frac{3}{2}(x + 4) + 11$ represent a linear function?

Complete the table for the function above. Does this show a linear function?

x	y
-4	
0	
2	
8	



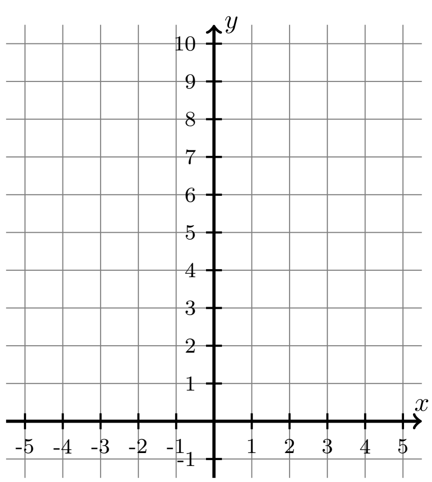
Plot the function above on the coordinate plane. Does this show a linear function?

Example

Does the rule $y = x^2$ represent a linear function?

Complete the table for the function above. Does this show a linear function?

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Plot the function above on the coordinate plane. Does this show a linear function?

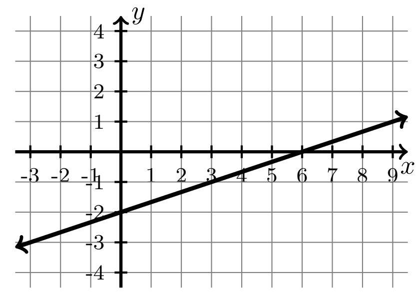
7.1 Intercepts

In a graph, an ______________ is a point where a function ____________ an ________.

An intercept on the x-axis is an ________________, and on the y-axis is a ________________.

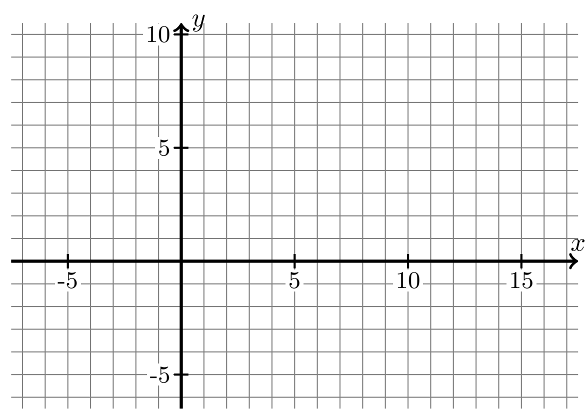
Example

State the intercepts of the graph.



Complete the table and plot for $y = -\dfrac{2x}{5} + 4$.

x	-5	0	5	10	15
y					



State the intercepts of the graph.

What do you notice? What do you wonder?

x-intercepts occur when __________. **y-intercepts occur when __________.**

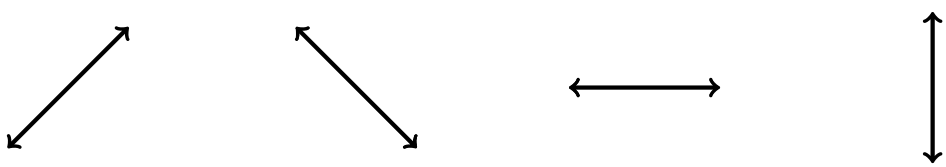
Example

Find the intercepts of the graph of $y = \frac{2}{3}x + 8$.

7.2 Slope

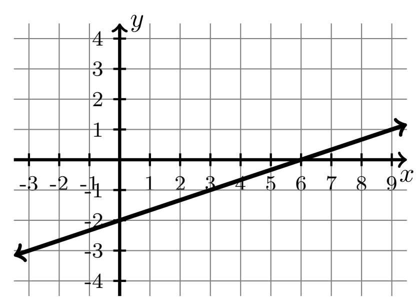
The __________ of a line is a measure of its _____________ and ________________. Slope is calculated as the ________ of the ________________________ to the __________________________ between two ___________ on the line.

The SLOPE of the graph of a LINEAR FUNCTION is identical to the function's ______________________.



Example

Calculate the slope of the line.

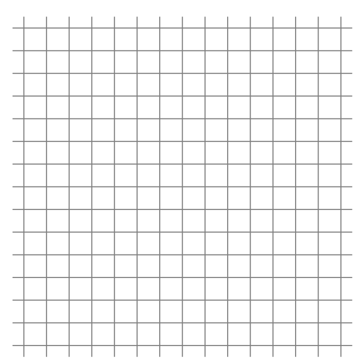


Example

Describe the direction of a line with slope $m = \frac{5}{2}$.

Describe the direction of a line with slope $m = -\frac{2}{3}$.

Draw an example of each slope on the grid provided.

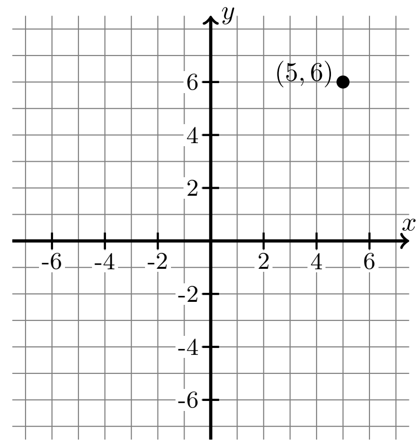


Example

Plot the line which passes through the point $(5, 6)$ with slope 2.

What are the intercepts of this line?

The point $(9, k)$ is also on the line. What is k?

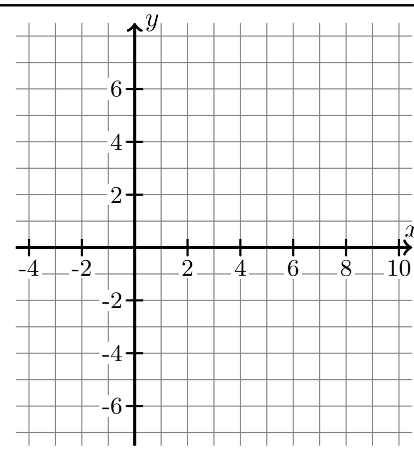


Example

What is the slope of the graph of $y = -\frac{1}{3}x + 2$?

What is the y-intercept? Plot it on the coordinate plane.

Plot a graph of the function by drawing a line from the y-intercept with the correct slope.



7.3 Slope-Intercept Form

We have already learned that:

- The ________ of a graph is the same as the ____________________ of the function.
- The ______________ is the point where the function's input is __________.
- The ______________ is the point where the function's output is __________.

SLOPE-INTERCEPT FORM

is the general form of a linear function __________________,

because m is the ________ of the graph

and $(0, b)$ is the ______________ of the graph.

A __________ is a type of graph which only shows the most important information of a function, such as ______________. A __________ must be ________, using a ________ for straight lines.

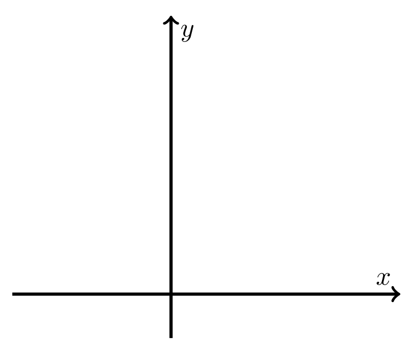
Example

Find the intercepts and the slope, then sketch the graph, of the function $y = -4x + 8$.

x-intercept: Solve $mx + b = 0$:

y-intercept:

Slope:

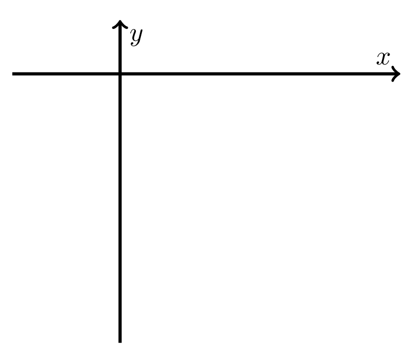


Find the intercepts and the slope, then sketch the graph, of the function $y = 2x - 6$.

x-intercept: Solve $mx + b = 0$:

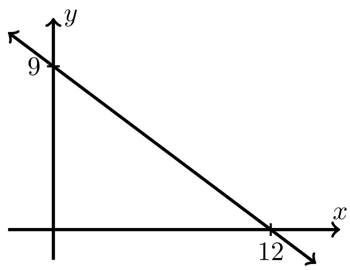
y-intercept:

Slope:

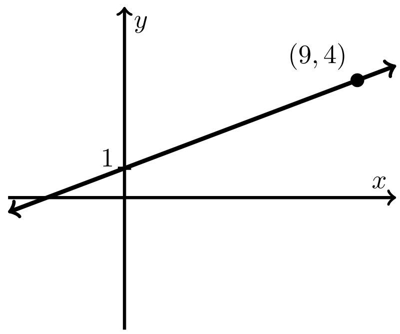


Example

Find the rule for the function in the sketch.

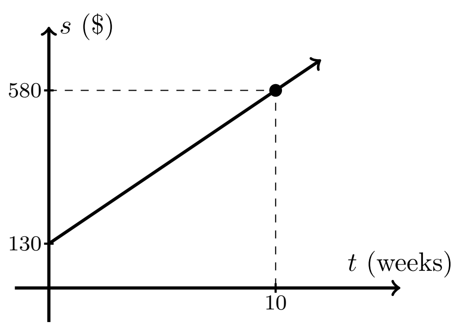


Find the rule for the function in the sketch, and find the location of the unlabeled x-intercept.



Example

Melanie has a savings account she is using to save up to buy a computer for \$850. Her savings balance since the start of the year is shown in the graph.



What does the independent variable represent?

What does the dependent variable represent?

What does the marked point represent?

What does the s-intercept represent?

Find the rule for the function representing Melanie's savings.

What is the slope of the graph? What does this represent?

When will Melanie's savings be enough for the computer?

7.4 Finding Linear Rules from Points

To write down a rule in ____________________ form, we need to know the _________ and the ________________. Sometimes, we need to use other _________ to find these.

Example

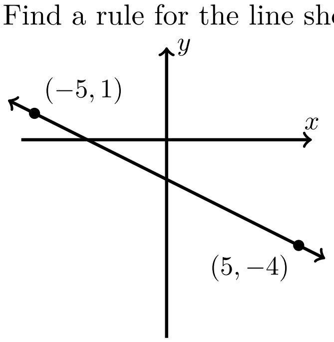
Find the y-intercept and the rule for each of the described lines.

Slope $m = -3$, passing through $(4, -1)$.

Slope $m = \frac{2}{5}$, x-intercept at $x = -10$.

Example

Find a rule for the line shown.



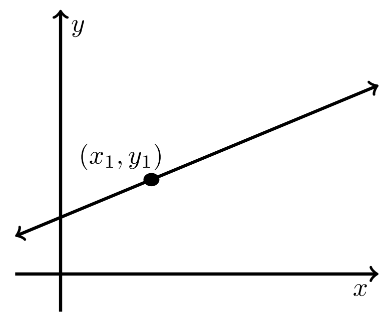
Find a rule for the line which passes through the points $(-1, 5)$ and $(7, 9)$.

Point-Slope Form

Suppose a ____________________ (x_1, y_1) is on a line. We can use (x, y) to represent ________________ on the line. The changes between the points are

Δx Δy

If the line has ________ m, then $\Delta y = m \cdot \Delta x$.



The POINT-SLOPE FORM of a line with slope m passing through (x_1, y_1) is

Example

a) Write rules for these lines in point-slope form.

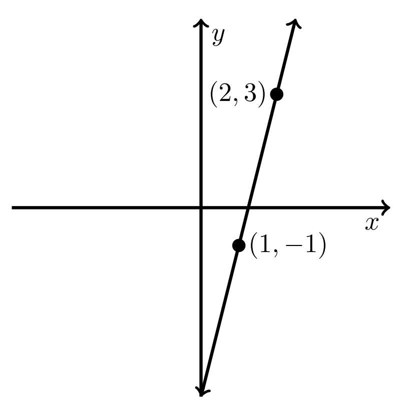
Slope $m = -2$, passing through $(-5, 7)$.

Slope $m = \frac{3}{4}$, passing through $(8, -2)$.

b) Write each rule in slope-intercept form.

Example

Find the rule for this line in both point-slope and slope-intercept forms.



7.5 Standard Form

The STANDARD FORM of the equation of a line is

- **Constants A, B and C are ____________, if possible.**
- **A is __________________.**
- **The equation is ______________, so A, B and C have no common ___________.**

Example

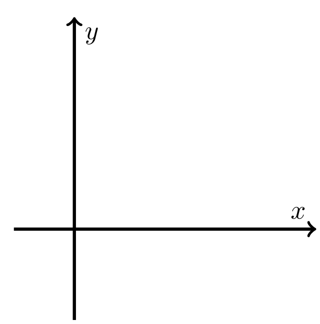
Do the points $(6, 5)$ and $(-2, 4)$ lie on the line $5x - 2y = 20$?

Remember that x-intercepts occur when __________, and y-intercepts occur when __________.

Example

Sketch a graph and find the slope of $x + 2y = 6$.

x-intercept: y-intercept: slope:



To find the _________ for an equation in standard form, we can use the _______________ to calculate it, or we can convert the equation to ____________________________.

Example

Check the results of the previous example by writing $x + 2y = 6$ in slope-intercept form.

Find the slope of $3x - 4y = 8$ by writing the rule in slope intercept form.

Example

Convert these linear functions to standard form.

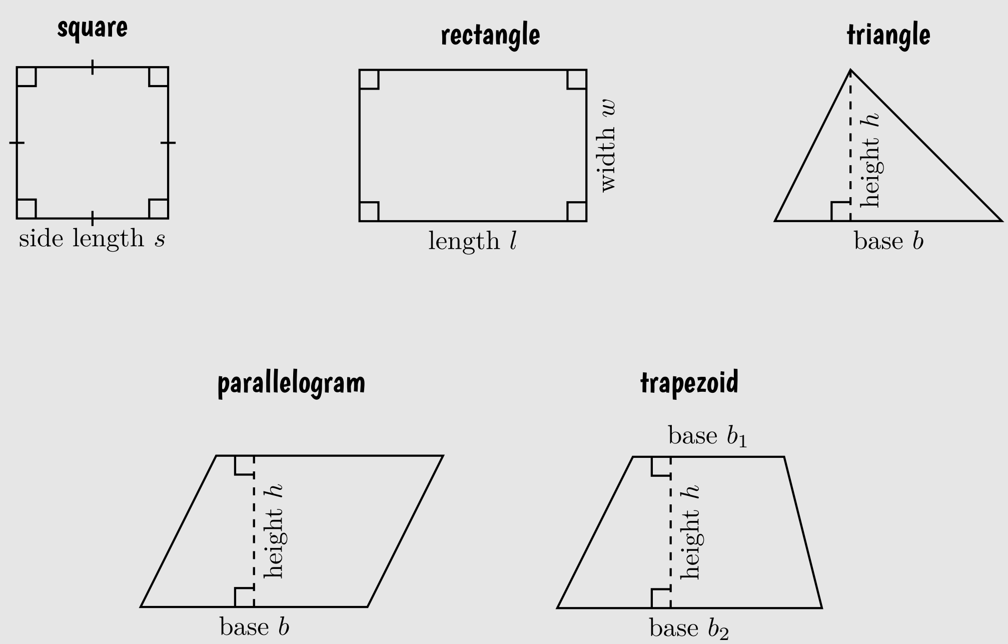
$y = \frac{2}{3}x - \frac{5}{6}$

$y = \frac{3}{4}x + \frac{5}{2}$

8.1 Perimeter and Area Review

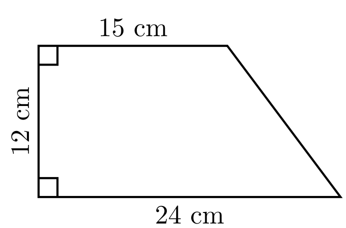
The ______________ of a closed figure is the total ___________ of its ______________. The _________ of a closed figure is a measure of the two-dimensional __________ contained in its ____________.

AREA of select two dimensional figures



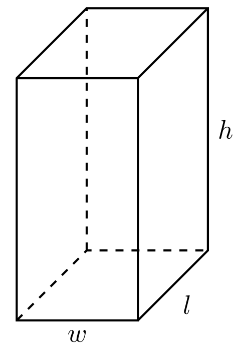
Example

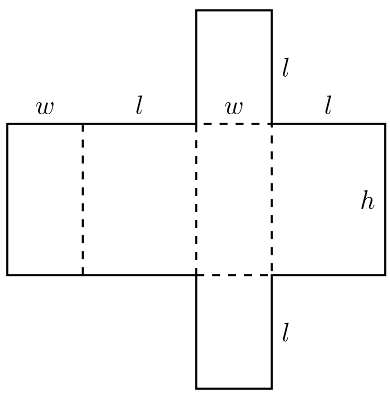
Find the area and the perimeter of the following figure.



8.2 Prism Surface Area

A __________ is a three-dimensional figure whose faces are two identical __________, connected on each edge by _______________ which are called the lateral faces. If the bases are also _______________, the shape is a _________________________. If each rectangle is a square, the shape is a _________.



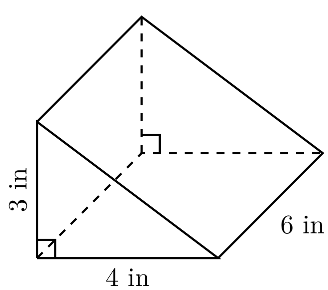


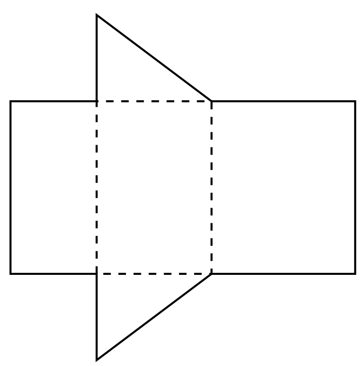
The ___________________ of a 3D shape is the sum of the __________ of its __________.

A _______ is a 2D representation of a 3D shape that forms the shape when __________ along its __________. The ____________________ of a 3D shape is the same as the _________ of its _______.

Example

Use the net to find the surface area of the rectangular prism.

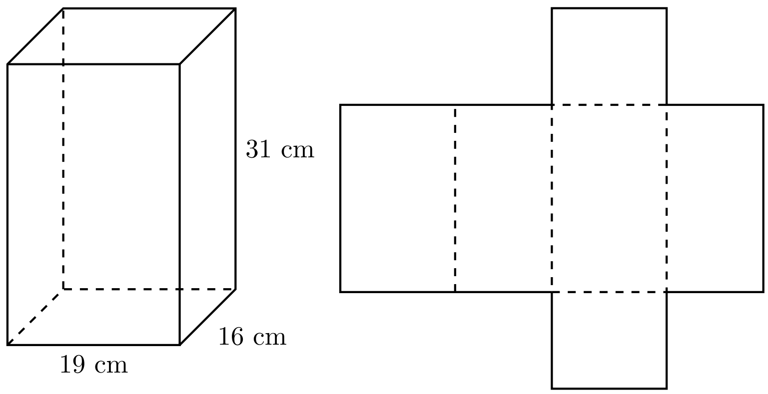




The SURFACE AREA OF A PRISM whose base has area B and perimeter P, and height is h

Example

Use the net to find the surface area of the rectangular prism.



The SURFACE AREA OF A RECTANGULAR with length l, width w, and height h

Example

Find the surface area of a cube with a side length of 11 inches.

8.3 Prism Volume

The ___________ of a 3D shape is a measure of the amount of three dimensional __________ it occupies. The volume of a __________ is the product of the area of its _________, and its ___________.

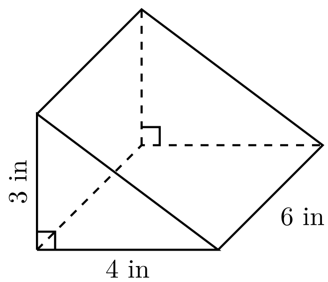
In a rectangular prism, the area of the _________ is the product of its ___________ and _________. This means that its ___________ can be found by multiplying its _________, __________, and ___________.

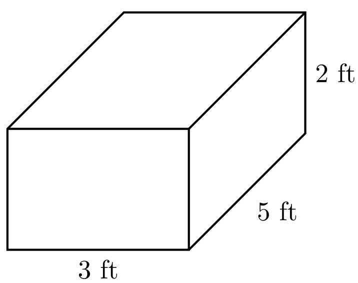
The VOLUME OF A PRISM with base area B and height h

The VOLUME OF A RECTANGULAR PRISM with length l, width w, and height h

Example

Find the volumes of the prisms shown.





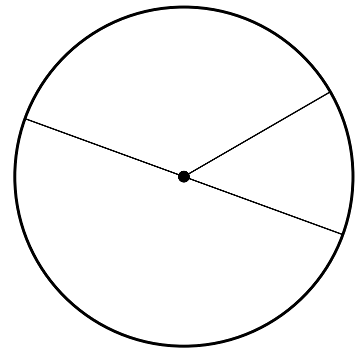
Example

A prism with a square base has a height of 5 mm and a volume of 80 mm^3. What is the width of the prism?

Example

A rectangular prism has a width of 4 in and a height of 7 in. The volume of the prism, in inches, is irrational. What can you say about the length of the prism?

8.4 Circles Review



A __________ is a 2D shape such that all its points are the same _____________ from its center.

A ___________ of a circle is a line segment between the center and a point on the circle. Its length is also called the __________.

A ______________ of a circle is a line segment between opposite points on the circle, which passes through the center. Its length, which is double the __________, is also called the ______________.

The _____________________ is the curved length around the circle.

π, the Greek letter ______, is the ratio of the _____________________ to the ______________ in every circle. Its value is an ______________ number which can be approximated using _______________.

The CIRCUMFERENCE C of a circle with radius r and diameter $d = 2r$

The AREA A of the interior of a circle with radius r

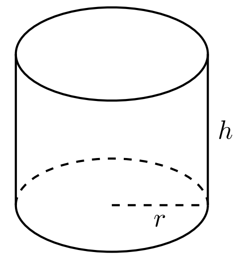
Example

Find the circumference and area of a circle whose diameter is 6 in.
Give answers exactly, and to two decimal places.

Example

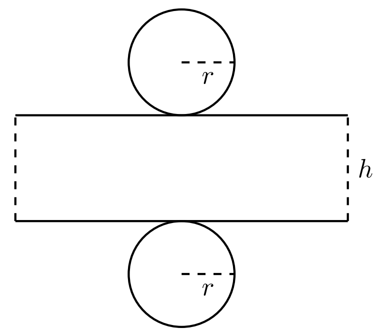
Find the area of a circle whose circumference is 24π cm.

8.5 Cylinder Surface Area



A ___________ is a 3D shape similar to a prism[1], with ___________ for the bases and a single ______________________ for the lateral surface.

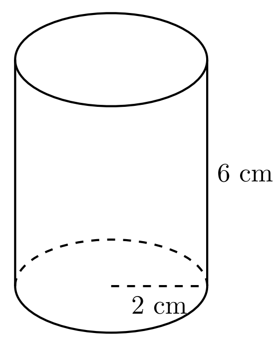
We can still use the surface area formula for prisms, _____________________. Since the bases are circles with radius r, we have the base area ______________ and perimeter (circumference) _______________.



The SURFACE AREA of a CYLINDER with base radius r and height h

Example

Find the surface area of the cylinder shown.

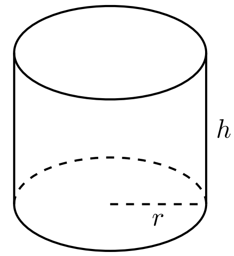


Example

A cylindrical tin cup has a diameter of 7 cm and a height of 10 cm. What is the area of the tin which forms the cup?

[1] Technically, a prism is a type of *polyhedron*, which means all the faces are flat polygons with straight edges. Because a circle is not a polygon, a cylinder is not a polyhedron, which means it can't be a prism.

8.6 Cylinder Volume



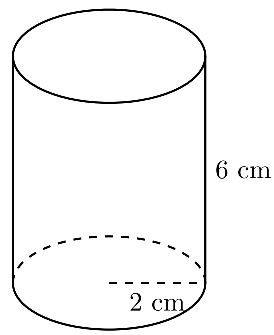
Recall that the volume of prism with base area B and height h is ______________. A ____________ is similar enough to a prism that this rule still holds.

We know that the base of a cylinder is a _________, and if its radius is r its base area is ______________. By substituting B, we get the formula for the ___________ of a ____________.

The VOLUME of a CYLINDER with base radius r and height h

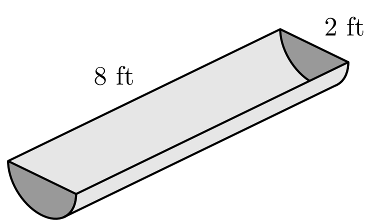
Example

Find the volume of the cylinder shown.



Example

Find the capacity of the water trough shown.



9.1 Measures of Central Tendency

A ____________ is a single measure which summarizes a characteristic of a collection of ________.

A _______________________________________ is a statistic which aims to represent a __________ value, or the ___________, of the data.

MEASURES OF CENTRAL TENDENCY

The __________ of a set of data is the ________ of the data values divided by the __________ (the number) of data values.

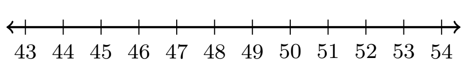
The ___________ of a set of data is the _________________ when the data are _____________ (for an odd count), or the mean of the _________________________ (for an even count).

The __________ of a set of data is _____________________ value in the data.

Another useful statistic is __________, which is a measure of the ___________ of the data, instead of the center. It is the ________________ between the ___________ and _____________ values.

Example

Complete the dot plot and calculate the mean, median, mode and range for the data:
46, 44, 47, 53, 45, 52, 45, 47, 49, 46, 45.



Example

In the first five basketball games of the season, Alex scores 8, 13, 6, 4 and 7 points. What are his mean and median scores?

In the sixth game, Alex scores 22 points. How does this change his mean and median scores?

Did including 22 in the data have a bigger effect on the mean or the median? Why?

Example

Karissa keeps track of the number of miles she runs each week. Over the last four weeks, she ran 6, 4, 8 and 6 miles respectively.

What is the mean distance Karissa ran each week? What is the median distance?

Without knowing how many miles Karissa runs in the fifth week, what could possibly happen to the mean and median?

9.2 Outliers

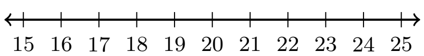
An ___________ is a value in a data set whose value is ____________ the range of values which could be expected from the rest of the data. This typically means ____________ are much ___________ or __________ than the rest of the data.

Outliers need to be carefully investigated, as they are sometimes the result of ___________. If an outlier exists, it's a good idea to find a ____________ its value doesn't fit the rest of the data.

Example

Complete the dot plot, and use it to identify any outliers for the following data:
22, 23, 25, 22, 15, 21

Find the mean and median of the data.



Find the mean and median with any outliers removed.

In general:

- Outliers can have a _________ effect on the _________.
- Outliers usually have a _________ effect, or even ______ effect, on the ___________.

9.3 Scatterplots and Lines of Best Fit

In statistics, a ____________ is a characteristic of a person or thing, which can have different values for each person or thing. The data for each person or thing is called an _________________. ____________________ consists of observations of _______ variables.

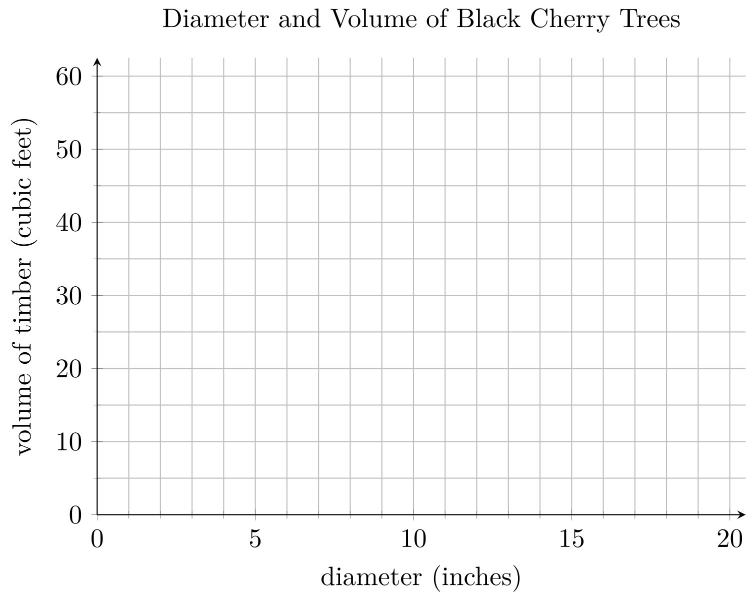
A _______________ is a plot which uses a coordinate plane to represent ___________________, with a ____________ on each ________. Each observation is plotted as a _________ on the plane.

Scatterplots should always include an appropriate _______, and _________ on each axis with appropriate _________.

Example

The table shows the diameter (in inches) and the volume (in cubic feet) of a selection of black cherry trees[1]. Represent the data on the coordinate plane as a scatterplot.

diameter (in)	volume (ft^3)
16.3	42.6
10.5	16.4
11.0	15.6
8.3	10.3
8.6	10.3
14.5	36.3
11.3	24.2
11.7	21.3
13.3	27.4
13.7	25.7
17.9	58.3
11.2	19.9



A ______________________ is a line we draw on a scatterplot so that it is as __________ as possible to each of the __________ on the scatterplot. The line shows the general __________ of the data.

In statistics, a __________ is a _____________ which approximates the _________________ between variables. The line of best fit represents a __________________ for our two variables.

For now we'll ______________________ the line of best fit. In high school, you'll use software or a calculator to do this more precisely.

Example

1. Draw the line of best fit for the previous scatterplot.
2. Estimate the volume of a black cherry tree with a diameter of 17 inches.
3. Estimate the rate of change of the volume of a black cherry tree with respect to its diameter.

[1] This is a subset of a dataset available in R, a programming language used by many statisticians. `https://search.r-project.org/R/refmans/datasets/html/trees.html`

10.1 Probabilities and Prediction

An ________________ is a random phenomenon whose ____________ is unknown until it occurs.

The ____________________ of an experiment is the set of all of its possible outcomes.

Example

State the sample space for each of the following.

1. The side shown on a flipped coin.
2. The value rolled on a standard 6-sided die.

An __________ is a subset of the sample space, or a collection of outcomes.

The _______________ of an event is a number between _____ and _____ inclusively which indicates how likely an experiment is to produce the __________. Probabilities can be written as __________________, ______________, or ______________.

If $P(A) = 0$, then event A is _______________.

If $P(A) = 1$, then event A is ____________.

If $P(A) = 0.5$, then event A is __________________ to occur or not occur.

Example

A fair coin is flipped. What is the probability of each of the following events?

- A: The coin lands heads up.
- B: The coin lands tails up.
- C: The coin lands either heads or tails up.
- D: The coin turns into a pony.

PROBABILITY of event A in sample space S with equally likely outcomes

Example

What is the probability that the value rolled on a 10-sided die is a prime number?

Is the number more likely or less likely to be prime than not prime?

Example

Two dice are rolled. Complete the table showing the sums of the possible dice rolls. Find the probabilities of the following events:

A: The sum of the two dice is 4.

B: The sum of the two dice is a multiple of 5.

Which sum is most likely to be rolled? What is its probability?

First Die Roll

Second Die Roll	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

10.2 Experimental Probability

Often it isn't possible to calculate the probabilities of events. Instead, we can ____________ an experiment many times, and use the outcomes to ______________ the probabilities of the events. These estimates are called ___________________________________.

A ________ is an individual performance of an experiment. Increasing the number of trials improves our confidence that the ___________________ probability is close to the _________ probability.

The EXPERIMENTAL (ESTIMATED) PROBABILITY of event A

Example

Janey is the goalkeeper for her soccer team. She keeps records for all the penalty kicks she defends. Last season, 21 penalty goals were scored against her, while she saved 9 attempts.

Estimate the percentage probability that Janey will save the next penalty kick against her.

Example

A bag contains an unknown mixture of colored marbles. One at a time, marbles are drawn randomly, then placed back in the bag. There were 7 blue marbles, 3 red marbles and 2 green marbles drawn.

Estimate the probability that the next marble is red.

Estimate the probability that the next marble is yellow.

Are there yellow marbles in the bag?

There are 48 marbles in the bag. Estimate the number of green marbles.

10.3 Independent and Dependent Events

Consider the probabilities of two (or possibly more) events. The events are called ________________ if the occurrence of one event ______________________ the probability of the other.

Events that are not independent are called ______________ events.

Example

Two dice are rolled. Let A be the event that the first die is even. Let B be the event that the second die is six.

What is $P(B)$?

Suppose we know that A occurs (the first die is even). What is $P(B)$ now?

Are A and B independent?

Example

One die is rolled. Let C be the event that the die is odd. Let D be the event that the die is five.

What is $P(D)$?

Suppose we know that C occurs (the die is odd). What is $P(D)$ now?

Are C and D independent?

The PROBABILITY of two INDEPENDENT EVENTS A and B both occuring is the ___________ of their individual probabilities

Example

Consider the two dice from the first example. What is the probability that the first is even at the same time the second is six?

10.4 Sampling Techniques

When using data, a ______________ is a collection of ______ the people or things in which we're interested. In practice, it may be too ____________ to collect data from the entire population. Instead, we only collect data from a ____________, which is a subset of the population.

Example

Identify the population and sample in each of the following.

1. A frozen foods factory chooses 10 pizzas to heat and test.
2. A pollster phones 500 voters to ask who they intend to vote for in the next election for Oklahoma governor.

A good sample should be ____________________ of the population, which means the data produces similar results. This means sample should be as _________ as is practical. This also means is should be a ____________________, meaning the members of the sample are chosen from the population ____________. A sample which is not ____________________ of the population is called a ____________________, or a ___________________.

Example

To determine the fitness of students at a middle school, 20 students are chosen to each run a mile. Are the following samples random or biased?

1. The principal makes an announcement asking for 20 volunteers.
2. 20 names are drawn from a hat with the names of every student in the school.
3. Each student is assigned a number, and a computer uses a random number generator to choose 20 numbers.
4. Mrs. Henley's sixth grade science class, which has 20 students.